Lecture 12
Grids

Euiseong Seo
(euiseong@skku.edu)
Grids

- Grids
  - Repetitive regular shapes that fill up the entire plane
  - Rectilinear, rectangular, triangular or hexagonal

- Many problems are related to grid representation
  - Map
  - VLSI layout
Rectilinear Grids

- Each cell is defined by horizontal and vertical lines
- Spacing between lines can be
  - Uniform
  - Non-uniform
- N-dimensional rectilinear grids
  - Each vertex touches 2N edges and $2^N$ cells
Representation and Traverse

- Grids are representable as a graph
  - (V, E)
  - V can be vertices or faces
  - E can be edges or neighboring relationships

- Traversal
  - Row major
  - Column major
  - Snake
  - Diagonal
Dual Graphs

- Two-dimensional arrays are the natural choice to represent planar rectangular grids

- $m[i][j]$ can be either
  - the $(i,j)$th vertex, or
  - the $(i,j)$th face

- $m[i][j][d]$
  - Adjacency representation for edge-weighted grids
  - $d$ ranges over four values (north, east, south and west)
Triangular Lattices

- A vertex needs to be assigned coordinates
  - Triangular coordinates
    - (x,y)
  - Geometrical coordinates
    - \((x_g, y_g) = (d(x_t + (y_t \cdot \cos(60))), dy \cdot \sin(60))\)
Hexagonal Lattices

- It is a subset of a triangular lattice
  - Remove every other vertex
- Interesting properties
  - It is the roundest polygon to fill the space
  - Per space/perimeter
    - A circle is the best
Hexagonal Lattices

- **Representation**
  - Deleting alternate vertices from a triangular lattice yields a hexagonal lattice
  - Hexagonal coordinates
  - Array coordinates
    - Conversion?
Plate Packing Problem

- How many plates can be packed in a \( l \times w \) sized box?

![Diagram of plate packing in a box](image)

12.3 Program Design Example: Plate Weight

A manufacturer of dinner plates seeks to enter the competitive campus dining hall market. Dining halls only buy plates in a single standard size, so they all stack neatly together. Students tend to break a lot of plates during food fights, so selling replacements can be a lucrative business. It is a very cost-sensitive market, however, since the administration gets quite tired of buying extra plates.

Our company seeks an edge in the market through its unique packaging technology. They exploit the fact that hexagonal lattices are denser than rectangular lattices by packaging the plates in \( l \times w \) boxes as shown in Figure 12.4(l). Each plate is \( r \) units in radius, and the lowest row contains exactly \( p = \lfloor \frac{w}{2r} \rfloor \) plates. Rows either always contain \( p \) plates or alternate between \( p \) and \( p-1 \) plates, depending on the relationship between \( w \) and \( r \). Each plate is assigned a unique identification number as shown in Figure 12.4(l). As many layers as possible are put in the box, subject to its length limit \( l \).

Management needs to know how many plates fit in a box of the given dimensions. They also need to know the maximum number of plates resting on top of any given plate to ensure that the packaging is strong enough to prevent breakage (if the post office will break the plates, who needs the students?). Your job is to identify which plate has the most stress and how many plates lie on top of it, as well as the total number of plates and layers in a box of given dimension.

---

Solution starts below———

The first problem is figuring how many rows of plates fit in the box under the hexagonal grid layout. This is one of the reasons why the Almighty invented trigonometry. The bottom row of plates rests on the floor of the box, so the lowest disk centers sit \( r \) units above the floor, where \( r \) is the plate radius. The vertical distance between successive rows of disks is \( 2r \sin(60^\circ) = 2r\sqrt{3}/2 \). We could simplify by canceling the 2's, but this obscures the origin of the formula.
Maja is a bee. She lives in a hive of hexagonal honeycombs with thousands of other bees. But Maja has a problem. Her friend Willi told her where she can meet him, but Willi (a male drone) and Maja (a female worker) have different coordinate systems:

- **Maja’s Coordinate System** — Maja (on left) flies directly to a special honeycomb using an advanced two-dimensional grid over the whole hive.

- **Willi’s Coordinate System** — Willi (on right) is less intelligent, and just walks around cells in clockwise order starting from 1 in the middle of the hive.

Help Maja to convert Willi’s system to hers. Write a program which for a given honeycomb number returns the coordinates in Maja’s system.
Bee Maja

Input
The input file contains one or more integers each standing on its own line. All honeycomb numbers are less than 100,000.

Output
Output the corresponding Maja coordinates for Willi’s numbers, with each coordinate pair on a separate line.

Sample Input

Sample Output

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 0</td>
</tr>
<tr>
<td>2</td>
<td>0 1</td>
</tr>
<tr>
<td>3</td>
<td>-1 1</td>
</tr>
<tr>
<td>4</td>
<td>-1 0</td>
</tr>
<tr>
<td>5</td>
<td>0 -1</td>
</tr>
</tbody>
</table>
Inspector Robostop is very angry. Last night, a bank was robbed and the robber escaped. As quickly as possible, all roads leading out of the city were blocked, making it impossible for the robber to escape. The inspector then asked everybody in the city to watch out for the robber, but the only messages he got were “We don’t see him.”

Robostop is determined to discover exactly how the robber escaped. He asks you to write a program which analyzes all the inspector’s information to find out where the robber was at any given time.

The city in which the bank was robbed has a rectangular shape. All roads leaving the city were blocked for a certain period of time $t$, during which several observations of the form “The robber isn’t in the rectangle $R_i$ at time $t_i$” were reported. Assuming that the robber can move at most one unit per time step, try to find the exact position of the robber at each time step.
Robbery

Input

The input file describes several robberies. The first line of each description consists of three numbers $W$, $H$, and $t$ ($1 \leq W, H, t \leq 100$), where $W$ is the width, $H$ the height of the city, and $t$ is the length of time during which the city is locked.

The next line contains a single integer $n$ ($0 \leq n \leq 100$), where $n$ is the number of messages the inspector received. The next $n$ lines each consist of five integers $t_i, L_i, T_i, R_i, B_i$, where $t_i$ is the time at which the observation has been made ($1 \leq t_i \leq t$), and $L_i, T_i, R_i, B_i$ are the left, top, right, and bottom, respectively, of the rectangular area which has been observed. The point $(1, 1)$ is the upper-left-hand corner, and $(W, H)$ is the lower-right-hand corner of the city. The messages mean that the robber was not in the given rectangle at time $t_i$.

The input is terminated by a test case starting with $W = H = t = 0$. This case should not be processed.
Output

For each robbery, output the line “Robbery #k:”, where k is the number of the robbery. Then, there are three possibilities:

If it is impossible that the robber is still in the city, output “The robber has escaped.”

In all other cases, assume that the robber is still in the city. Output one line of the form “Time step τ: The robber has been at x,y.” for each time step in which the exact location can be deduced, and x and y are the column and row, respectively, of the robber in time step τ. Output these lines ordered by time τ.

If nothing can be deduced, output the line “Nothing known.” and hope that the inspector does not get even angrier.

Print a blank line after each processed case.
Sample Input

4 4 5
4
1 1 1 4 3
1 1 1 3 4
4 1 1 3 4
4 4 2 4 4
10 10 3
1
2 1 1 10 10
0 0 0

Sample Output

Robbery #1:
Time step 1: The robber has been at 4,4.
Time step 2: The robber has been at 4,3.
Time step 3: The robber has been at 4,2.
Time step 4: The robber has been at 4,1.

Robbery #2:
The robber has escaped.