Lecture 8
Backtracking

Euiseong Seo
(euiseong@skku.edu)

Backtracking

- Systematic method to iterate through all the possible configurations of a search space
- General algorithm/technique which must be customized for each individual application

General Backtracking

- Solution is a vector $a = (a_1, a_2, \ldots a_n)$
- a_i is selected from a finite ordered set S_i
  1. At each step, start from a given partial solution $a = (a_1, a_2, \ldots a_k)$
  2. Try to extend it by adding another element at the end
  3. After extending it, test whether what we have so far is a solution
     1. If not, check whether the partial solution is still potentially extendible to some complete solution
        1. If so, recur and continue
        2. If not, we delete the last element from a and try another possibility for that position, if one exists
     2. If so, finishes

Implementation

```
bool finished = FALSE;                  /* found all solutions yet? */

backtrack(int a[], int k, data input)
{
        int c[MAXCANDIDATES];           /* candidates for next position */
        int ncandidates;                /* next position candidate count */
        int i;                          /* counter */

        if (is_a_solution(a,k,input))
                process_solution(a,k,input);
        else {
                k = k+1;
                construct_candidates(a,k,input,c,&ncandidates);
                for (i=0; i<ncandidates; i++) {
                        a[k] = c[i];
                        backtrack(a,k,input);
                        if (finished) return;   /* terminate early */
                }
        }
}
```

Application-Specific Routines

- is_a_solution(a, k, input)
- construct_candidates(a, k, input, c, ncandidate)
- process_solution(a, k)

Constructing All Subsets

- Using the general backtracking algorithm
- S_k = (true, false)
- *a* is a solution whenever k >= n

Constructing All Subsets

```
is_a_solution(int a[], int k, int n)
{
        return (k == n);                /* is k == n? */
}

construct_candidates(int a[], int k, int n, int c[], int *ncandidates)
{
        c[0] = TRUE;
        c[1] = FALSE;
        *ncandidates = 2;
}

process_solution(int a[], int k)
{
        int i;                          /* counter */

        printf("{");
        for (i=1; i<=k; i++)
                if (a[i] == TRUE) printf(" %d",i);

        printf(" }\n");
}
```

Constructing All Subsets

```
generate_subsets(int n)
{
        int a[NMAX];                        /* solution vector */

        backtrack(a,0,n);
}
```

Constructing All Permutations

- $S_k = \{1,2,\ldots n\} - a$
  - To avoid repeating permutation elements
- *a* is a solution whenever k = n

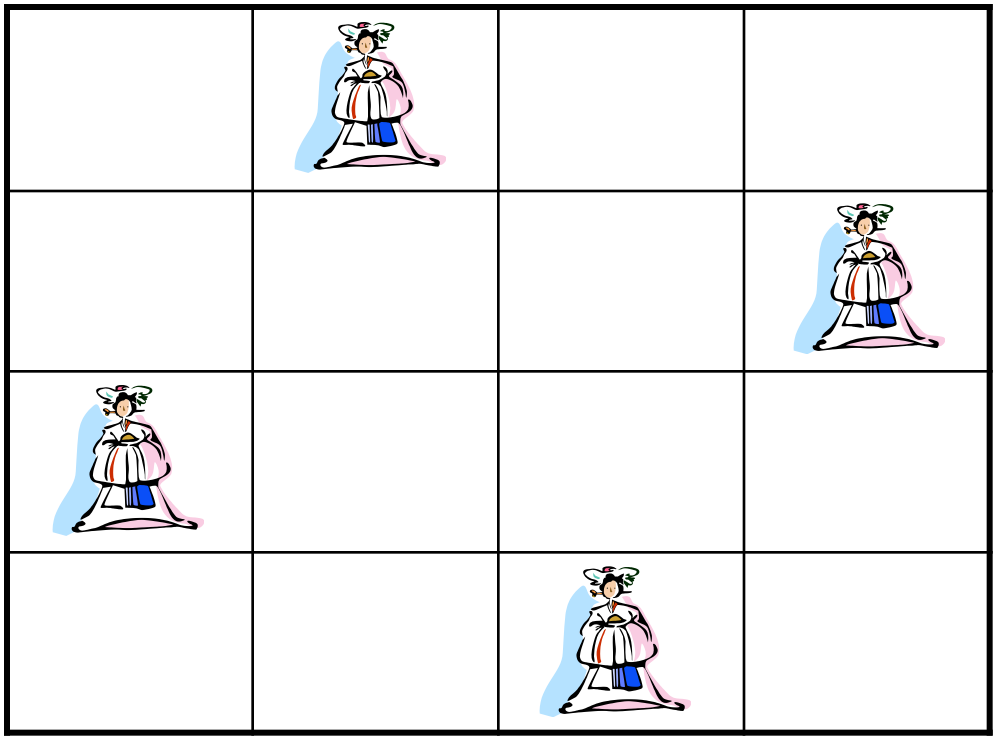
```
construct_candidates(int a[], int k, int n, int c[], int *ncandidates)
{
        int i;                          /* counter */
        bool in_perm[NMAX];             /* who is in the permutation? */

        for (i=1; i<NMAX; i++) in_perm[i] = FALSE;
        for (i=0; i<k; i++) in_perm[ a[i] ] = TRUE;

        *ncandidates = 0;
        for (i=1; i<=n; i++)
                if (in_perm[i] == FALSE) {
                        c[ *ncandidates] = i;
                        *ncandidates = *ncandidates + 1;
                }
}
```

Constructing All Permutations

```
process_solution(int a[], int k)
{
        int i;                                  /* counter */

        for (i=1; i<=k; i++) printf(" %d",a[i]);
        printf("\n");
}

is_a_solution(int a[], int k, int n)
{
        return (k == n);
}

generate_permutations(int n)
{
        int a[NMAX];                            /* solution vector */

        backtrack(a,0,n);
}
```

N-Queens Problem

4-Queens Problem



N-Queens Problem

- Applying backtracking when n = 8
  - S_k = (queen, no queen)
  - 2^{64} search space
- Think a_i is *i-th* queen's position
  - 64^8
  - Exploit the fact that all queens are identical

N-Queens Problem

- Only a single queen can exist in a row
- A row must have a queen
- We can save search space by utilizing these facts
  - Pruning
  - 8! of complexity

N-Queens Problem

```
construct_candidates(int a[], int k, int n, int c[], int *ncandidates)
{
      int i,j;                          /* counters */
      bool legal_move;                  /* might the move be legal? */

      *ncandidates = 0;
      for (i=1; i<=n; i++) {
          legal_move = TRUE;
          for (j=1; j<k; j++) {
                  if (abs((k)-j) == abs(i-a[j]))  /* diagonal threat */
                          legal_move = FALSE;
                  if (i == a[j])                  /* column threat */
                          legal_move = FALSE;
          }
          if (legal_move == TRUE) {
                  c[*ncandidates] = i;
                  *ncandidates = *ncandidates + 1;
          }
      }
}
```

15 Puzzle Problem

Tug of War

A tug of war is being arranged for the office picnic. The picnickers must be fairly divided into two teams. Every person must be on one team or the other, the number of people on the two teams must not differ by more than one, and the total weight of the people on each team should be as nearly equal as possible.

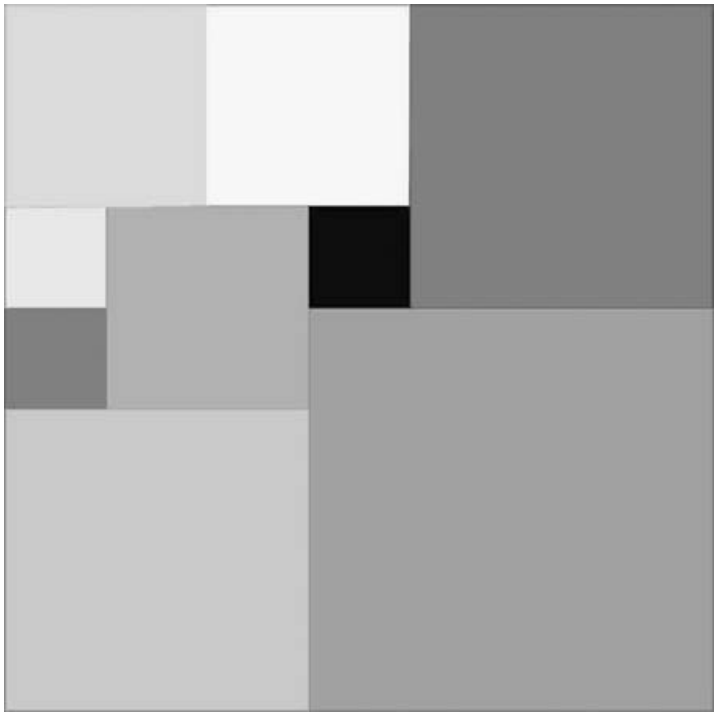
Sample Input

```
1

3
100
90
200
```

Sample Output

```
190 200
```

Bigger Square Please



You have unlimited squares, of which side length ranges from 1 to N-1
How can you build a square of side length N?